EXHIBIT A

IN THE UNITED STATES DISTRICT COURT
FOR THE EASTERN DISTRICT OF TEXAS
MARSHALL DIVISION

CYWEE GROUP, LTD., PLAINTIFF,	)
VS.	) CIVIL ACTION NO.: 2:17-CV-00140-RWS-RSP
SAMSUNG ELECTRONICS CO. LTD, AND SAMSUNG ELECTRONICS AMERICA, INC., DEFENDANTS.	)

ORAL AND VIDEOTAPED DEPOSITION OF

M. RAY MERCER, PhD

03/09/2108

ORAL AND VIDEOTAPED DEPOSITION OF M. RAY MERCER, PhD, produced as a witness at the instance of the Plaintiff, and duly sworn, was taken in the above-styled and numbered cause on March 9, 2018, 2018, from 8:21 a.m. to 4:36 p.m., before Kelly Bryant, CSR in and for the State of Texas, reported by machine shorthand, at Shore Chan, 901 Main Street, Dallas, pursuant to the Federal Rules of Civil Procedure and the provisions stated on the record or attached hereto.

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A P P E A R A N C E S

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I N D E X

E X H I B I T S

EXHIBIT	DESCRIPTION	PAGE
1	Asserted Claims	5
2	Joint Claim Construction	5
3	Declaration of Mercer	5
4	Patent '438	42
5	Patent '978	42
6	Inertial frame of reference	64
7	Extended Kalman filter	143
8	Optimal State Estimation	163

P R O C E E D I N G S

VIDEOGRAPHER: And we're going on the record in the videotaped deposition of Dr. M. Ray Mercer. Today's date is March 9, 2018. The time is 8:21 a.m.

At this time, counsel please state your appearances for the record and the court reporter will swear in the witness.

MR. RAFILSON: Ari Rafilson, on behalf of CyWee Group, and with me is Dr. Nicholas Gans, one of our experts in the case.

MS. BRANN: Liza Brann, on behalf of Samsung, and with me is my associate Steve Mosley.

M. RAY MERCER, PhD,

having been first duly sworn, testified as follows:

Show direct examination

DIRECT EXAMINATION

BY MR. RAFILSON:

Q. Welcome, Dr. Mercer.

Would you state your name for the record?

A. Yes, my full name is Melvin Ray Mercer.

Q. Okay. And just as a housekeeping matter, before we get started, I understand from our discussion before we got started, that you've brought some materials that you would like to refer to during your

Q. (BY MR. RAFILSON) -- by the extended Kalman filters?
A. Terms that carry that variable to values in the Kalman filter.
Q. How many papers have you written on Kalman filters?
MS. BRANN: Objection, form.
A. None.
Q. (BY MR. RAFILSON) None. How many patents do you have on patent -- on Kalman filters?
A. I think you already asked that question.
Q. No, I asked -- I'm sorry, I asked papers first.
A. No, you asked me papers, but early on you asked me about patents.
Yeah, if you want to ask me the question, again, I guess that's okay. I just want to let you know you were asking the same question twice, I believe.
Q. Okay.
A. And by the way, if you look at the record and I'm wrong, ask me and I'll be happy -- happy to answer.
Q. So just to confirm, you don't have any patents on Kalman filters?
A. No, none of -- none of the patents that I have relate to Kalman filters. That's absolutely right.
But they certainly do relate to the same

kind of ideas and the same kind of objectives that are achieved by Kalman filters. It's just that in the cases I was working on that -- those were not -- that Kalman filter was the optimal implementation.

There was another way of doing things.

Q. And when you say "the same types of objectives," what do you mean?

A. So, for example, if we go to the example that comes to my mind, that's closest to, essentially, the -- the topics in this litigation, with respect to inertial guidance and with respect to rockets, when -- when rockets are -- are fired, they go through a stress period.

And it is possible that the electronics that are critical to the operation of the rockets will not tolerate the stress. They will not work properly.

And so in order to reduce that probability or increase the confidence that you have and the ability of the electronics to endure those stresses, stresses are imposed prior to the actual launch of the vehicle, often prior to the construction of the vehicle.

Q. And how does that relate to a Kalman filter?

A. It relates to a Kalman filter because the right way to handle that problem, in that particular case, was definitely not a Kalman filter.

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Q. Okay.
A. But it -- but it used very similar ideas. It's just it was not optimal.
Q. Have you ever implemented an extended Kalman filter?
A. No. By the way, the term never appears in -- in the patent note specifications.
Q. Dr. Mercer, while I appreciate that, I would ask you to keep your answers to the questions I'm asking.
A. Fair enough.
Q. Your -- your counsel will have an opportunity to ask you questions during -- during redirect?
A. I'd -- I'd like to answer your questions so well, my counsel doesn't have to ask questions.
Q. Okay. Let's turn back to Exhibit 3.
A. I have that.
Q. Your Declaration. So you had talked earlier and said that you spent approximately 110 to 140 hours on this case, as a whole, correct?
A. That's correct.
Q. About how much time would you say you spent working on this Declaration?
A. Oh, I don't know. I don't keep up with that. In fact, I don't -- I have not the slightest idea. I

the beginning of media number three. The time is 11:36.

Q. (BY MR. RAFILSON) Thank you. Dr. Mercer, I would refer you back to Exhibit 4, the '438 patent.

A. I have that.

Q. Okay. And I want to also refer you to -- well, actually, never mind that.

Let me refer you to Figure 1.

A. I have Figure 1.

Q. Okay. And we established earlier that 110 is defined -- or described as a pointing device, right?

A. In the specif -- in the pointing device in the specification, yes.

Q. Correct. And does pointing device 110 in Figure 1 appear to be a rigid object?

A. I -- I actually, when I -- there's nothing in this figure that tells me that, but when I hear the word "pointing device," I think about the pointing devices that I've used in the past, and I can't remember one that was not rigid.

Q. Okay. So does device 110 appear to be a rigid object?

A. I already answered that question.

And the answer is, if -- if you interpret 110 as a pointer device, then, yes, it's rigid. If I interpret it just looking at this figure, there's not

Q. Right. And is it your understanding from reviewing the patents that the patent involves tracking motion of a USB stick plugged into the side of the computer?
A. I don't recall that.
Q. Okay.
A. Actually, that -- I'm sorry. I should say, no, I don't recall any discussion of the USB port, and whether the USB -- the memory stick.
I don't recall whether that was ever discussed, or whether the fact that it was rigid or nonrigid, I don't recall that that was an issue. I just wanted to point out, I was asking -- answering your question fully and completely.
Q. And I appreciate that.
A. Sure. And you deserve it.
Q. Are you aware of any other disclosure of a nonrigid object in the figures before you?
A. Okay. If you're talking about the key assembly, the keyboard assembly...
Q. Well, actually, let's ignore the -- let's ignore the PC and USB key shown in Figure 5.
A. Okay. Okay. Okay.
Q. Just to shortcut --
A. Sure. Okay. That's great.

Q. -- your answer.
A. I don't see any other object in figures 1 Through 9 of the '438 patent, except those that I've already identified for you that look to be nonrigid. They all -- the remainder looks to be rigid.
Q. Okay. And I'm not asking you to -- okay.
Do you recall during your preparation for this case having found any disclosures in the specification that disclose a nonrigid body?
A. Oh, well, you mean explicitly or implicitly?
Q. First, let's start with explicitly.
A. I don't remember one way or the other.
Q. So you don't recall any explicit disclosure of a nonrigid body in the '438 patent?
A. No, I didn't say that.
I said, I don't remember one way or the other. It may be there and I have forgotten it. So this is an answer to the question that I don't know the answer to your question.
Q. Okay.
A. But I'll read the specification, if you want, and then I can answer your question.
Q. Okay. So let's turn to paragraph 46, or actually 45 of your Declaration.
A. Okay. I have my Declaration and my

Wikipedia...
Q. Yeah. Okay.
A. So I'm looking --
Q. I'm referring you to Exhibit 7.
A. Okay.
Q. You already said that you didn't recall seeing the equations in the equations 5 Through 11 in the '438 patent.
A. Okay. I've --
Q. Essentially --
A. I've been bouncing back and forth between the '438 and this one.
Q. Well, I'm asking the same question with respect to discreet time predict and update equations that you see in Exhibit 7.
A. Right. And what is that question?
Q. The question is: Have you seen any of these -- are you familiar with any of these equations?
A. I don't -- I don't have an immediate recollection of seeing these equations using these terms.
Even if I had seen something that was isomorphic, I'm not sure I would recognize it, because I -- the context for those variables may well be completely different from here.

So the answer is, gosh, I can't -- I can't say.

Q. Okay. Let's look at the innovation or measurement residual equation in figure -- in Exhibit 7.

A. Okay.

Q. And would you or would you not say that that is equivalent to equation 8 from the '438 patent?

A. Well, the first difference that I see in 8, is that the first term -- by the way, yeah, as expressing K of T given T minus one in the innovation of measurement residual, I don't see -- and there are two terms.

And both of those -- okay. So actually there's -- there's one term here, and it's compound because it's H of X of T when we look at 8, but there's no -- there's no compound notation with respect to the innovation or measurement residual at all.

It's not compound. You don't have to go and resolve something that's inside a bracket, parenthesis, to figure out what it is that's in there so no, those are not -- those are not isomorphic.

Maybe repeat your question and I can answer it.

Q. What about the -- I want to refer you -- do you see the formulation section on that same page of Exhibit 7?

A. I'm sorry. Formulation, okay.
Q. Yeah. Okay. You see the second formula Z subK equals H, left paren, X of K, and goes on from there?
A. If you're referring to the K -- the equation that says Z of K equals H of K -- I'm sorry, H of X subK, plus B sub...
Q. SubK?
A. It is subK.
Q. Yeah. Would you say that that equation is, essentially, the same as equation 8 from the '438 patent?
A. No. It definitely is not. On the left hand side -- I'm sorry. Wait a minute. I looked at the wrong -- okay. Let me go back.
Okay. Certainly, equation 8 seems to be written in terms of time and, therefore, one of ordinary skill in the art would be more inclined to think about it as this being continuous. That's the first thing I notice.
And then you said 8, and the second one, correct?
Q. Yeah, the Z subK --
A. SubK. Okay.
Q. -- equation that you just read.
A. Okay.

Q. Or the formulation section.
A. The same thing with the T. Actually, 8 is written in conditional form, Z of T given T minus one. So that's a conditional formulation. There's no conditional formulation in Z subK equals H of XK plus BK.
Okay. And the H -- KH here is again compound. It's H of X of T given T minus one, whereas, here it's just V subK.
So, no, in fact, it was worse than that. Equation 8 -- equation 8 on the right-hand side has one compound variable Z, subK on the right-hand side has two variables that are being added.
So, no, there -- that's -- that's -- that's not the same formulation.
Q. Is it similar?
MS. BRANN: Objection, form.
Q. (BY MR. RAFILSON) To a person of ordinary skill in the art, is it essentially the same?
MS. BRANN: Objection, form.
A. Yeah, my answer to that is I don't think so.
In fact, I'm -- let me tell you, if a student came to me and said, you know, I wrote down the answer and it is similar to this, rather than what I was expecting, that would not be similar to me.

They would not win the similar argument unless they got the gimme.

Q. Let's turn to Exhibit 7.

Do you see the innovation or residual covariance equation?

A. I do.

Q. And is it your -- and is that equivalent to equation 9 from the '438 patent?

A. Certainly, the left hand variables are different. They are not -- not explicitly -- there's -- there's conditional probability as what's being expressed on 9.

I don't think there's a conditional probability on the innovation or residual covariance. So they're definitely not matched up that way. To the extent that you interpret the subscripts as being the same as the -- the variables that are inside the open and closed parenthesis, and that first term -- 9 -- 9 HKP -- you have to assume, again from the way these were written, I think one skilled in the art would understand because of the parenthesis those are continuous statements, and the -- and the one from Wikipedia is discreet, but it's closer.

The real problem is that it's P of K given K minus 1 on the left-hand side of Wikipedia, oh, okay,

and Z of TT minus one -- also these things are -- these things differ in the sense that Wikipedia chooses capital letters for the variables, and equation 9, oh, I misstated.

Those are -- those are capital letters. So they're both in -- probably in a different domain, probably not a temporal domain. They're probably in some sort of response domain.

We're talking about 9, correct?

Q. We are talking about 9.

A. I'm sorry, we're comparing the innovation or residual covariance, correct, with equation 9?

Q. Yes.

A. Yeah. The -- the major difference that I see here is that it looks pretty clear to me that Wikipedia is a discreet formulation. It's ambiguous with respect to equation 9, whether it's discreet --

Q. When you say, "it's ambiguous" -- are you talking about -- oh, you're talking about the '438 patent?

You're talking about equation 9 in the '438 patent, right, when you discussed...

A. Yeah -- yeah, I'm sorry. Absolutely. I apologize. Okay. So the -- the difference that I notice is that in the Wikipedia paper, this is clearly

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something that's done that's not continuous. It's discreet.

And in the '438, it's ambiguous whether it's discreet or continuous.

Q. So to you, the fact that -- that equation 9 references TNT minus one does not connote that the equation is discreet; is that -- is that correct.

I don't want to misstate your testimony.

A. Okay. Sure. That's a good question.

So now, you're referring -- I assume you're referring to equation 9, right?

Q. Yes.

A. Okay.

Q. Because you indicated that it didn't look discreet, and so I'm --

A. Right. On the --

Q. -- I'm just referring to --

A. Right. Because --

Q. TNT minus 1.

A. If you notice the variable here is -- is like time. Do you see it is a T? The -- the subscript is a T, right?

On the -- yeah, so the index or the -- or the continuous variable is T over in the 438. The K there is a discreet value corresponding to a discreet.

It doesn't have to be time, necessarily. It can be a sample or some such thing as that. I can't really say about that.

Q. Okay. Let's turn the, let's go to page 3 of Exhibit 7.

MS. BRANN: Which page is that, Ari?

MR. RAFILSON: It's the page that starts F subK equals.

THE WITNESS: Okay. I'm sorry. Would you start the question over?

Q. (BY MR. RAFILSON) Would you turn to page 3 of Exhibit 7. That's the Wikipedia article.

A. Okay. Page 3 of the Wikipedia article. This is one. This is two.

Do you see, Higher-order extended Kalman filterS?

Q. Yes.

A. Okay.

Q. Do you see the equation at the top of the page?

A. Yes.

Q. Is that the same as equation 6 of the '438 patent, asymmetrically?

A. This is the -- this is -- starts over on Wikipedia and it says XK equals F of X -- K minus one, comma, one U of K minus one, comma WK minus one.

Is that the formula are you talking about or are you talking about another formula?

Q. Hold on a moment. It doesn't start out XK. It starts out FK equals.

A. FK.

Q. It's the very top of the page under, Higher-Order extended Kalman filter.

A. Okay.

Q. It's the top equation.

A. Got it. Got it. Got it. Okay. Okay.

The same argument about continuous versus variable. I -- X of T minus one. Oh, T minus one. No. I don't think -- let me -- partial F with respect to X T minus one. In this particular case -- let's see, this particular case, on the '438, we're working -- okay.

So we have -- if you notice F subX equals the partial F, open parenthesis, XT minus one comma UT, with the partial -- with the partial taken with respect to XT minus one.

Over here -- I guess that's okay, partial with respect to X of -- no. No. No, that's not -- let's see, no. The partial in one case is a function only of XT minus one. That's in the case of the -- the '438 patent.

And -- and this one, this is a really --

this is a -- actually, this is a very strange notation, because here it's saying...

Q. When you're referring to "here," are you referring to Exhibit 7 or the '438?

A. Exhibit 7, which is the Wikipedia.

If you notice it is X hat so K minus one given K minus one. That is a strange thing to say. If K minus one is K minus one, you don't have to put the given K minus one. I don't even know what that connotes, just a strange notation or sloppy notation.

Okay. And we're looking now at equation 6 -- equation 6, yes. So also if you notice over here --

Q. When you say "over here"?

A. I'm sorry, when you -- when you look at the Wikipedia, you notice that it's given -- and now, this is X hat K minus one, given K minus one.

So that hat over the top of it doesn't appear over here in equation 6. I think the chances are pretty good that the carrot mean estimate. There's nothing to indicate on the other side that -- that that's estimated.

Does that make sense? Wait a minute.

Q. So --

A. I'm comparing -- I'm comparing F with F. These things run together after a while. (Indiscernible)

Q. So you're saying there's -- there's no carrot in the -- in the -- in the Wikipedia article?

A. There is a carrot with respect to F in the Wikipedia article.

Q. Right. But not with respect to the X, which is the denominator?

A. Well, absolutely. That's -- that's X where you take the derivative with X when you set it to the estimate, something about given FX one, and then given K minus one.

Honestly I -- I find the Wikipedia -- that -- that terminology is -- I don't understand it, but -- but also in one case, there's an estimate and the other case there's not.

Q. Okay. And referring to the equation underneath that, the one underneath S subK of H subK equation --

A. Right. Right.

Q. -- is that substantively identical to equation 10 of the '438 patent?

MS. BRANN: Objection, form.

A. Well, I think this is definitely strange, because if you notice the variable -- the left hand variable in 6 and 7 are, in fact, F of X and F subU.

But here they're F and H and F subU. I don't know what F subU -- I don't know whether F subU

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has any relationship to X of U or not, but I -- I know for isomorphism, you have to find the same thing on both sides, generally, because these can be -- can be, obviously, entered into other equations to do the calculations.

So the fact that the left hand variables are different here means to me they're not isomorphic, because in this isomorphism -- for all of isomorphisms for -- for the '438, the two left hand variables are the same.

And in Wikipedia, the left hand variables on top is F subK and the bottom is H subK, and then also the point of evaluation in the case of the '438, is it time X of T minus one and -- okay, and -- and here it's X hat given subK given K.

And there was one other thing. Let me see what that was. Oh -- oh, in order -- and you also -- in order to avoid another problem, you have to make the assumption that the capital F subU, and then you talk about the partial with respect to the lower case F, whereas in Wikipedia, you see the capital H of K and then the partial of lower case H.

That's -- that's okay, assuming that in both cases, upper and lower case mean the same, except perhaps a different transform, like one in time and one

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in frequency domain, or something like that.

But you'd also have to -- you'd have to recognize that to even get as close as I was after I gave you all the (indiscernible) --

THE WITNESS: And I don't want to, John, to interrupt your flow or anything, but it was a very good lunch and I would like to try -- bio break at your convenience.

MR. RAFILSON: Sure. We can take a break now.

THE WITNESS: Thank you very much.

VIDEOGRAPHER: And we're going off the record at 2:52.

(Break)

VIDEOGRAPHER: We're back on the record for the beginning of media number five. The time is 3:05.

MR. RAFILSON: Dr. Mercer, I'm going to give you what will be marked as Exhibit 8.

(Plaintiff's Exhibit No. 8 marked)

Q. (BY MR. RAFILSON) And, Dr. Mercer, do you agree that this is a publication -- this is an excerpt of a publication entitled, Optimal State Estimation?

A. Well, I'm -- I'm sure that you're representing that this came from a document that was printed by Wiley-Intersciences, called Optimal State Estimation,

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and you got the copyright here. So you've the date.

And then -- and then you wanted me to particularly look at 1 page of that -- or 2 pages.

Q. Are you familiar with this work -- with this piece of work, Optimal State Estimation?

A. I -- I really don't know the answer to that question, because I've been looking at -- at Kalman -- Kalman publications, and I looked at a lot of them.

And so, certainly, I would not be able to remember every equation that was written every place, but I have been looking at Kalman -- I've been looking at Kalman's writing, and so I -- I don't know whether I looked at this one or not.

Oh, is there a date on this, by the way? No, this is the book date. This is 2006. It's copyright 2006, but we don't know that Kalman actually wrote this in 2006. It might be something that was copied from something he had written back in 1961. True?

The only reason I'm asking is I kind of know something about the dates I'm looking at. And if you're going to tell me it's a 2006 document, Kalman wrote this for the first time in 2006, I can guarantee you I haven't seen it.

Q. I'm not going to represent when the document

was written, but certainly it has a copyright page that speaks for itself.

A. That -- that speaks for when it was published, but it does not speak about when it was -- I mean, that's -- let me just explain that, that's my understanding.

MR. RAFILSON: Actually, I have to admit that there is a missing page from this exhibit. So we're going to have to go on break for a minute while I print off the missing page.

VIDEOGRAPHER: Going off the record?

MR. RAFILSON: Yeah, let's go off the record for just a moment.

VIDEOGRAPHER: Off the record at 3:09.

(Off the record)

VIDEOGRAPHER: We're back on the record. The time is 3:10.

MR. RAFILSON: Dr. Mercer, the previous version of Exhibit 8 I gave you was missing page 409. I'm going to hand that to you right -- right now.

THE WITNESS: Okay.

Do you have what you need, Kelly?

MR. RAFILSON: If you want to staple it to the other exhibit for convenience, you can. You don't have to.

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THE WITNESS: Okay.

MR. RAFILSON: Liza, here is your copy.

Q. (BY MR. RAFILSON) Dr. Mercer, I'd like you to turn to page 409.

A. I have that.

Q. And have you seen any of these equations before?

A. I think I have testified that I reviewed several common publications. I've also testified in the past that I don't have recollection of every equation that I see, and so I don't know one way or the other.

Q. Okay. Just a couple of questions on this one. I'd refer you to the first equation under step 1.

The system and measurement equations are given as follows, and this is for the discreet time extended Kalman filter.

A. Yes.

Q. Okay.

A. And -- and they're numbered 1344, four equations all of which I would assume is group number 1344.

Q. Right. Right. And you've already testified that you don't recall if you've seen these specific equation before.

A. True.

Q. Is -- is the first equation equivalent to equation 5 of the '438 patent?

MS. BRANN: Objection, form.

A. Okay. The first thing I note is, do you remember before I had conjectured that the formulations that were described by Wikipedia were discreet, and that's why those Ks were at the bottom of the -- of the variables, the subKs at the bottom of the variables.

If you look just above this, notice that it said discreet time extended Kalman filter. So that conjecture was correct. So that observation contrast, 5 may be discreet, but it also can be continuous.

So that certainly means they're not equivalent.

Q. (BY MR. RAFILSON) Okay. And is the second equation equivalent to equation 8 of the '438 patent? So this is the second equation of Exhibit 8 to equation -- I'm sorry.

The second equation on page 409 of Exhibit 8, yes, and we're comparing that to equation 8 of the '438 patent.

MS. BRANN: Objection, form.

A. Okay. So the first thing is that equation 8, the variables are evaluated and the value of TT minus one, and at -- the second thing that we see on

Wikipedia --

Q. (BY MR. RAFILSON) Well, for clarification, this isn't Wikipedia.

Oh, are you referring to Exhibit 7, or are you referring to Exhibit 8? If Exhibit 8 is not Wikipedia?

A. My apologize. You're right. I -- I've changed context. Actually, there's probably some mistakes in the prior testimony too, but after -- after you gave me this new document --

Q. Yes.

A. -- I was thinking that you made another copy from Wikipedia, but now I remember it's not. It's from a different book.

And so if you want to repeat the questions or --

THE WITNESS: If you want, I'll just be happy, I guess, with the advice of counsel to speculate, when I said Wikipedia with respect to this document, I really meant --

MR. RAFILSON: That's fine.

THE WITNESS: Is that okay with everybody?

MS. BRANN: Yeah.

THE WITNESS: It will save you some time.

MR. RAFILSON: Okay.

A. Okay. Now, we're back to 8 and we're looking at 8 in the second equation. Okay. Here we have Z subT at T given T1.

So this is an evaluation at time T minus one, as I understand it, very well could be a discreet time, but it also could be a continuous time.

In contrast Y subK in the case of this document is clearly discreet.

Q. Well, when you say -- I'm sorry to interrupt. When you say "this document," would you more clearly refer to, say, the patent or the exhibit, just to distinguish.

A. Okay.

Q. Just to distinguish.

A. That's -- that's -- that's very reasonable. That's very reasonable. So I can -- I can do that, and I will do a better job, okay.

And so let me start -- why don't you ask the question and I will restart -- and I will restart my answer.

Q. Okay. So I referred you to the second equation under it's at -- it's under one, the system and measurement equations listed for discreet time extended filter.

And that is in Exhibit 8, and that's in --

it's labeled Section 13.44 and this is the second equation there that reads Y subK equals H subK and continues on from there.

And I'm asking you if that is substantially the same as equation 8 of the '438 patent?

MS. BRANN: Objection, form.

A. So my first answer with respect to the '438 patent, the left hand variable, which is being calculated, is a function of time, and it's not clear. This might be evaluated at discreet times or it could be evaluated at continuous time.

This notation would not discriminate between those two, but in contrast this Exhibit 8 version clearly is done discreetly. So that's the first thing I find that is different.

The second thing that I find is different is the time setting, and so in the case of '438 patent, the -- the time when we're looking is time T minus one, both for the thing we're looking at and for the result.

With respect to Exhibit 8, this -- this is clearly discreet and it's not at T minus one, and it's not at K minus one. It's at time K and, to me, these really mean something different in the sense that in one case, you're relying on something that had been simple -- sampled at least one time frame early, where -- and

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that's in the case of the '438 patent.
But in the case of the Exhibit 8, this indicates that that sample is taken and it is immediately available, and it does, in no way, reflect the fact that there's been a period of time, a wait period of one time unit, whatever that my be.
That may sound like a trivial difference, but in many cases, it's not and it really has to do with causality.
Q. (BY MR. RAFILSON) Okay. What about -- I want you to refer to the equation -- see Section 3D on page 409 of Exhibit 8.
A. I have 3D.
Q. Right. Do you see the first equation there. It's listed K subK equals P subK and moves on from there?
A. I do.
Q. Right. And if -- hold on one moment.
Is that substantively the same as equation 9 of the '438 patent?
MS. BRANN: Objection, form.
A. Yeah.
By the way, I want to stipulate that as I answer these questions, I'm answering these questions in terms of the particular equation that you cite here, and

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not in terms of the total disclosures of the two documents, because if I tried to do that, it would take a lot longer.

Q. (BY MR. RAFILSON) Okay.

A. Okay. So in the case of Document 8, there is an -- if you -- if you read this directly and we're talking about the first line, agreed? This says X subK equals, and then the first term that we see is P subK, and then there's a straight horizontal line.

Do you see that?

Q. Yes.

A. Okay. To me, I'm not sure what that means and I think there are two possible interpretations. The first one is it may mean P minus, and P minus can have all kinds of, you know, meanings, but it differentiates it from P.

On the other hand, since there's a line over K, this could be P at K bar. One of skill in the art would not be able to revolve that question without additional information.

Since that problem does not exist over here with respect to --to -- with respect to equation 9 --

Q. Of the '438 patent?

A. -- of the '438 patent, certainly, and that's -- that's not a match. PK --

Now, the next thing, if you notice in equation 9, the order is HXP of X minus one given X minus one, but over here, you have a multiplication by the transform. Let me be sure that's right. Yeah.

You're multiplying P minus K, whatever that is, by H transform K. That transformation on H is not reflected in equation 9 of the '438 patent, but it is -- it does show up in different -- in -- in different -- different orders, and it is a transform.

So in this particular case, we take HX times P. In this case over here, we take P minus, whatever that is, times H transform.

Now, P is evaluated at -- explicitly evaluated at X minus one in the 438. K is just K, so it has the same time shift that -- that we discussed earlier.

Q. Okay.

A. There's -- there's -- oh, wow, this is -- I mean -- maybe I'm looking at the wrong thing.

You said D -- with respect to Exhibit 8, you said D and you said the first line.

Q. Yes.

A. Okay. And then you also said equation 9?

Q. Yes.

A. You're sure?

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Q. Yes.

A. Okay. Well, I think the differences here are so obvious to the casual observer that it's not necessary for me to render any further opinion.

Notice that R1 appears on the right-hand side equation 9 and we already know from Dr. Laviola that that's an artificial noise that's introduced in the process.

We don't have anything that I can see here, unless -- rather than just pure R, there's an MK, RK, MK transform -- even then they're not multiplied in the same way.

Well, first they're added, and there's no addition here. So that -- those are so different that the casual observer, I think, will see the difference.

Q. Okay. So I would refer you back to your Declaration, and I'm going to refer you to --

THE WITNESS: Do you happen to have a paperclip? Probably not?

So now is a good time to staple this if you want to keep the exhibit together.

MR. RAFILSON: I have a binder clip, if you --

THE WITNESS: Whatever -- it's your exhibit. You can do whatever you want to with it.

IN THE UNITED STATES DISTRICT COURT
FOR THE EASTERN DISTRICT OF TEXAS
MARSHALL DIVISION

CYWEE GROUP, LTD.,)
)
)
PLAINTIFF,)
)
VS.) CIVIL ACTION NO.:
) 2:17-CV-00140-RWS-RSP
)
SAMSUNG ELECTRONICS CO.)
LTD, AND SAMSUNG)
ELECTRONICS AMERICA,)
INC.,)
)
)
DEFENDANTS.)

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DEPOSITION OF M. RAY MERCER, PhD

03/09/2108

I, Kelly Bryant, Certified Shorthand Reporter in and for the State of Texas, hereby certify to the following:

That the foregoing deposition of M. RAY MERCER, PhD, the witness, hereinbefore named was, at the time named, taken by me in stenograph on 03/09/2108 having been first duly cautioned and sworn to tell the truth, the whole truth, and nothing but the truth, and the same

were thereafter reduced to typewriting by me or under my direction.

The charge for the completed deposition is $_________, due from Plaintiff; By agreement of counsel, the deposition transcript, has been sent to MR. ARI RAFLINSON, Shore Chan Depumpto, LLP, 901 Main Street, Suite 3300, Dallas, Texas 75202, arafilson@shorechan.com on _____________ for review and signature within 30 days and if any corrections returned are attached hereto;

I further certify that I am neither counsel for, related to, nor employed by any of the parties in the action in which this proceeding was taken, and further that I am not financially or otherwise interested in the outcome of the action.

GIVEN UNDER my hand of office on the March 12, 2018.

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Extended Kalman filter

In estimation theory, the **extended Kalman filter (EKF)** is the nonlinear version of the Kalman filter which linearizes about an estimate of the current mean and covariance. In the case of well defined transition models, the EKF has been considered[1] the *de facto* standard in the theory of nonlinear state estimation, navigation systems and GPS.[2]

Contents

History

The papers establishing the mathematical foundations of Kalman type filters were published between 1959 and 1961.[3][4][5] The Kalman Filter is the optimal estimate for *linear* system models with additive independent white noise in both the transition and the measurement systems. Unfortunately, in engineering, most systems are *nonlinear*, so some attempt was immediately made to apply this filtering method to nonlinear systems. Most of this work was done at NASA Ames.[6][7] The EKF adapted techniques from calculus, namely multivariate Taylor Series expansions, to linearize a model about a working point. If the system model (as described below) is not

well known or is inaccurate, then Monte Carlo methods, especially particle filters, are employed for estimation. Monte Carlo techniques predate the existence of the EKF but are more computationally expensive for any moderately dimensioned state-space.

Formulation

In the extended Kalman filter, the state transition and observation models don't need to be linear functions of the state but may instead be differentiable functions.

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}, \boldsymbol{u}_k) + \boldsymbol{w}_k$$

$$\boldsymbol{z}_k = h(\boldsymbol{x}_k) + \boldsymbol{v}_k$$

Where $\mathbf{w}_k$ and $\mathbf{v}_k$ are the process and observation noises which are both assumed to be zero mean multivariate Gaussian noises with covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$ respectively. $\mathbf{u}_k$ is the control vector.

The function f can be used to compute the predicted state from the previous estimate and similarly the function h can be used to compute the predicted measurement from the predicted state. However, f and h cannot be applied to the covariance directly. Instead a matrix of partial derivatives (the Jacobian) is computed.

At each time step, the Jacobian is evaluated with current predicted states. These matrices can be used in the Kalman filter equations. This process essentially linearizes the non-linear function around the current estimate.

Consider Kalman Filter for notational remarks.

Discrete-time predict and update equations

Predict

Predicted state estimate	$\hat{\boldsymbol{x}}_{k\|k-1} = f(\hat{\boldsymbol{x}}_{k-1\|k-1}, \boldsymbol{u}_k)$
Predicted covariance estimate	$\boldsymbol{P}_{k\|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1\|k-1} \boldsymbol{F}_k^\top + \boldsymbol{Q}_k$

Update

Innovation or measurement residual	$\tilde{\boldsymbol{y}}_k = \boldsymbol{z}_k - h(\hat{\boldsymbol{x}}_{k\|k-1})$
Innovation (or residual) covariance	$\boldsymbol{S}_k = \boldsymbol{H}_k \boldsymbol{P}_{k\|k-1} \boldsymbol{H}_k^\top + \boldsymbol{R}_k$
Near-optimal Kalman gain	$\boldsymbol{K}_k = \boldsymbol{P}_{k\|k-1} \boldsymbol{H}_k^\top \boldsymbol{S}_k^{-1}$
Updated state estimate	$\hat{\boldsymbol{x}}_{k\|k} = \hat{\boldsymbol{x}}_{k\|k-1} + \boldsymbol{K}_k \tilde{\boldsymbol{y}}_k$
Updated covariance estimate	$\boldsymbol{P}_{k\|k} = (\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k) \boldsymbol{P}_{k\|k-1}$

where the state transition and observation matrices are defined to be the following Jacobians

$$\boldsymbol{F}_k = \left.\frac{\partial f}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_k}$$

$$\boldsymbol{H}_k = \left.\frac{\partial h}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}_{k|k-1}}$$

Higher-order extended Kalman filters

The above recursion is a first-order extended Kalman filter (EKF). Higher order EKFs may be obtained by retaining more terms of the Taylor series expansions. For example, second and third order EKFs have been described.[6] However, higher order EKFs tend to only provide performance benefits when the measurement noise is small.

Non-additive noise formulation and equations

The typical formulation of the **EKF** involves the assumption of additive process and measurement noise. This assumption, however, is not necessary for **EKF** implementation.[9] Instead, consider a more general system of the form:

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1}, \boldsymbol{w}_{k-1})$$

$$\boldsymbol{z}_k = h(\boldsymbol{x}_k, \boldsymbol{v}_k)$$

Where $\mathbf{w}_k$ and $\mathbf{v}_k$ are the process and observation noises which are both assumed to be zero mean multivariate Gaussian noises with covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$ respectively. Then the covariance prediction and innovation equations become

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1|k-1}\boldsymbol{F}_{k-1}^{\top} + \boldsymbol{L}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{L}_{k-1}^{T}$$

$$\boldsymbol{S}_k = \boldsymbol{H}_k\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^{\top} + \boldsymbol{M}_k\boldsymbol{R}_k\boldsymbol{M}_k^{T}$$

where the matrices $\boldsymbol{L}_{k-1}$ and $\boldsymbol{M}_k$ are Jacobian matrices:

$$\boldsymbol{L}_{k-1} = \left.\frac{\partial f}{\partial \boldsymbol{w}}\right|_{\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1}}$$

$$\boldsymbol{M}_k = \left.\frac{\partial h}{\partial \boldsymbol{v}}\right|_{\hat{\boldsymbol{x}}_{k|k-1}}$$

The predicted state estimate and measurement residual are evaluated at the mean of the process and measurement noise terms, which is assumed to be zero. Otherwise, the non-additive noise formulation is implemented in the same manner as the additive noise **EKF**.

Continuous-time extended Kalman filter

Model

$$\begin{aligned} \dot{\mathbf{x}}(t) &= f\big(\mathbf{x}(t), \mathbf{u}(t)\big) + \mathbf{w}(t) & \mathbf{w}(t) &\sim \mathcal{N}\big(\mathbf{0}, \mathbf{Q}(t)\big) \\ \mathbf{z}(t) &= h\big(\mathbf{x}(t)\big) + \mathbf{v}(t) & \mathbf{v}(t) &\sim \mathcal{N}\big(\mathbf{0}, \mathbf{R}(t)\big) \end{aligned}$$

Initialize

$$\hat{\mathbf{x}}(t_0) = E\big[\mathbf{x}(t_0)\big], \mathbf{P}(t_0) = Var\big[\mathbf{x}(t_0)\big]$$

Predict-Update

$$\begin{aligned} \dot{\hat{\mathbf{x}}}(t) &= f\big(\hat{\mathbf{x}}(t), \mathbf{u}(t)\big) + \mathbf{K}(t)\Big(\mathbf{z}(t) - h\big(\hat{\mathbf{x}}(t)\big)\Big) \\ \dot{\mathbf{P}}(t) &= \mathbf{F}(t)\mathbf{P}(t) + \mathbf{P}(t)\mathbf{F}(t)^{\top} - \mathbf{K}(t)\mathbf{H}(t)\mathbf{P}(t) + \mathbf{Q}(t) \\ \mathbf{K}(t) &= \mathbf{P}(t)\mathbf{H}(t)^{\top}\mathbf{R}(t)^{-1} \\ \mathbf{F}(t) &= \left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(t), \mathbf{u}(t)} \\ \mathbf{H}(t) &= \left.\frac{\partial h}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(t)} \end{aligned}$$

Unlike discrete-time extended Kalman filter, the prediction and update steps are coupled in continuous-time extended Kalman filter.[10]

Discrete-time measurements

Most physical systems are represented as continuous-time models while discrete-time measurements are frequently taken for state estimation via a digital processor. Therefore, the system model and measurement model are given by

$$\begin{aligned} \dot{\mathbf{x}}(t) &= f\big(\mathbf{x}(t), \mathbf{u}(t)\big) + \mathbf{w}(t) & \mathbf{w}(t) &\sim \mathcal{N}\big(\mathbf{0}, \mathbf{Q}(t)\big) \\ \mathbf{z}_k &= h(\mathbf{x}_k) + \mathbf{v}_k & \mathbf{v}_k &\sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k) \end{aligned}$$

where $\mathbf{x}_k = \mathbf{x}(t_k$.

Initialize

$$\hat{\mathbf{x}}_{0|0} = E\big[\mathbf{x}(t_0)\big], \mathbf{P}_{0|0} = Var\big[\mathbf{x}(t_0)\big]$$

Predict

$$\text{solve} \begin{cases} \dot{\hat{\mathbf{x}}}(t) = f\big(\hat{\mathbf{x}}(t), \mathbf{u}(t)\big) \\ \dot{\mathbf{P}}(t) = \mathbf{F}(t)\mathbf{P}(t) + \mathbf{P}(t)\mathbf{F}(t)^{\top} + \mathbf{Q}(t) \end{cases} \quad \text{with} \begin{cases} \hat{\mathbf{x}}(t_{k-1}) = \hat{\mathbf{x}}_{k-1|k-1} \\ \mathbf{P}(t_{k-1}) = \mathbf{P}_{k-1|k-1} \end{cases}$$

$$\Rightarrow \begin{cases} \hat{\mathbf{x}}_{k|k-1} = \hat{\mathbf{x}}(t_k) \\ \mathbf{P}_{k|k-1} = \mathbf{P}(t_k) \end{cases}$$

where

$$\mathbf{F}(t) = \left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(t),\mathbf{u}(t)}$$

Update

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^{\top}\left(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^{\top} + \mathbf{R}_k\right)^{-1}$$
$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\left(\mathbf{z}_k - h(\hat{\mathbf{x}}_{k|k-1})\right)$$
$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k|k-1}$$

where

$$\mathbf{H}_k = \left.\frac{\partial h}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k|k-1}}$$

The update equations are identical to those of discrete-time extended Kalman filter.

Disadvantages of the extended Kalman filter

Unlike its linear counterpart, the extended Kalman filter in general is *not* an optimal estimator (of course it is optimal if the measurement and the state transition model are both linear, as in that case the extended Kalman filter is identical to the regular one). In addition, if the initial estimate of the state is wrong, or if the process is modeled incorrectly, the filter may quickly diverge, owing to its linearization. Another problem with the extended Kalman filter is that the estimated covariance matrix tends to underestimate the true covariance matrix and therefore risks becoming inconsistent in the statistical sense without the addition of "stabilising noise" [11] .

Having stated this, the extended Kalman filter can give reasonable performance, and is arguably the *de facto* standard in navigation systems and GPS.

Modifications

Iterated extended Kalman filter

The iterated extended Kalman filter improves the linearization of the extended Kalman filter by recursively modifying the centre point of the Taylor expansion. This reduces the linearization error at the cost of increased computational requirements.

Robust extended Kalman filters

The extended Kalman filter arises by linearizing the signal model about the current state estimate and using the linear Kalman filter to predict the next estimate. This attempts to produce a locally optimal filter, however, it is not necessarily stable because the solutions of the underlying Riccati equation are not guaranteed to be positive definite. One way of improving performance is the faux algebraic Riccati technique [12] which trades off optimality for stability. The familiar structure of the extended Kalman filter is retained but stability is achieved by selecting a positive definite solution to a faux algebraic Riccati equation for the gain design.

Another way of improving extended Kalman filter performance is to employ the H-infinity results from robust control. Robust filters are obtained by adding a positive definite term to the design Riccati equation.[13] The additional term is parametrized by a scalar which the designer may tweak to achieve a trade-off between mean-square-error and peak error performance criteria.

Invariant extended Kalman filter

The invariant extended Kalman filter (IEKF) is a modified version of the EKF for nonlinear systems possessing symmetries (or *invariances*). It combines the advantages of both the EKF and the recently introduced symmetry-preserving filters. Instead of using a linear correction term based on a linear output error, the IEKF uses a geometrically adapted correction term based on an invariant output error; in the same way the gain matrix is not updated from a linear state error, but from an invariant state error. The main benefit is that the gain and covariance equations converge to constant values on a much bigger set of trajectories than equilibrium points as it is the case for the EKF, which results in a better convergence of the estimation.

Unscented Kalman filters

A nonlinear Kalman filter which shows promise as an improvement over the EKF is the unscented Kalman filter (UKF). In the UKF, the probability density is approximated by a deterministic sampling of points which represent the underlying distribution as a Gaussian. The nonlinear transformation of these points are intended to be an estimation of the posterior distribution, the moments of which can then be derived from the transformed samples. The transformation is known as the unscented transform. The UKF tends to be more robust and more accurate than the EKF in its estimation of error in all the directions.

> "The extended Kalman filter (EKF) is probably the most widely used estimation algorithm for nonlinear systems. However, more than 35 years of experience in the estimation community has shown that is difficult to implement, difficult to tune, and only reliable for systems that are almost linear on the time scale of the updates. Many of these difficulties arise from its use of linearization."[1]

A 2012 paper includes simulation results which suggest that some published variants of the UKF fail to be as accurate as the Second Order Extended Kalman Filter (SOEKF), called also the augmented Kalman filter.[14] The SOEKF predates the UKF by approximately 35 years with the moment dynamics first described by Bass et al.[15] The difficulty in implementing any Kalman-type filters for nonlinear state transitions stems from the numerical stability issues required for precision,[16] however the UKF does not escape this difficulty in that it uses linearization as well, namely linear regression. The stability issues for the UKF generally stem from the numerical approximation to the square root of the covariance matrix, whereas the stability issues for both the EKF and the SOEKF stem from possible issues in the Taylor Series approximation along the trajectory.

See also

- Kalman filter
- Ensemble Kalman filter
- Fast Kalman filter
- Invariant extended Kalman filter
- Moving horizon estimation
- Particle filter
- Unscented Kalman filter

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External links

- Position estimation of a differential-wheel robot based on odometry and landmarks (http://correll.cs.colorado.edu/?p=1464)

Optimal State Estimation

Kalman, H_∞, and Nonlinear Approaches

Dan Simon
Cleveland State University

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$$\begin{aligned}
x_k &= f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) + \left.\frac{\partial f_{k-1}}{\partial x}\right|_{\hat{x}_{k-1}^+} (x_{k-1} - \hat{x}_{k-1}^+) + \left.\frac{\partial f_{k-1}}{\partial w}\right|_{\hat{x}_{k-1}^+} w_{k-1} \\
&= f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) + F_{k-1}(x_{k-1} - \hat{x}_{k-1}^+) + L_{k-1}w_{k-1} \\
&= F_{k-1}x_{k-1} + \left[f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) - F_{k-1}\hat{x}_{k-1}^+\right] + L_{k-1}w_{k-1} \\
&= F_{k-1}x_{k-1} + \tilde{u}_{k-1} + \tilde{w}_{k-1} \qquad (13.39)
\end{aligned}$$

F_{k-1} and L_{k-1} are defined by the above equation. The known signal $\tilde{u}_k$ and the noise signal $\tilde{w}_k$ are defined as follows:

$$\begin{aligned}
\tilde{u}_k &= f_k(\hat{x}_k^+, u_k, 0) - F_k\hat{x}_k^+ \\
\tilde{w}_k &\sim (0, L_kQ_kL_k^T) \qquad (13.40)
\end{aligned}$$

We linearize the measurement equation around $x_k = \hat{x}_k^-$ and $v_k = 0$ to obtain

$$\begin{aligned}
y_k &= h_k(\hat{x}_k^-, 0) + \left.\frac{\partial h_k}{\partial x}\right|_{\hat{x}_k^-} (x_k - \hat{x}_k^-) + \left.\frac{\partial h_k}{\partial v}\right|_{\hat{x}_k^-} v_k \\
&= h_k(\hat{x}_k^-, 0) + H_k(x_k - \hat{x}_k^-) + M_kv_k \\
&= H_kx_k + \left[h_k(\hat{x}_k^-, 0) - H_k\hat{x}_k^-\right] + M_kv_k \\
&= H_kx_k + z_k + \tilde{v}_k \qquad (13.41)
\end{aligned}$$

H_k and M_k are defined by the above equation. The known signal z_k and the noise signal $\tilde{v}_k$ are defined as

$$\begin{aligned}
z_k &= h_k(\hat{x}_k^-, 0) - H_k\hat{x}_k^- \\
\tilde{v}_k &\sim (0, M_kR_kM_k^T) \qquad (13.42)
\end{aligned}$$

We have a linear state-space system in Equation (13.39) and a linear measurement in Equation (13.41). That means we can use the standard Kalman filter equations to estimate the state. This results in the following equations for the discrete-time extended Kalman filter.

$$\begin{aligned}
P_k^- &= F_{k-1}P_{k-1}^+F_{k-1}^T + L_{k-1}Q_{k-1}L_{k-1}^T \\
K_k &= P_k^-H_k^T(H_kP_k^-H_k^T + M_kR_kM_k^T)^{-1} \\
\hat{x}_k^- &= f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) \\
z_k &= h_k(\hat{x}_k^-, 0) - H_k\hat{x}_k^- \\
\hat{x}_k^+ &= \hat{x}_k^- + K_k(y_k - H_k\hat{x}_k^- - z_k) \\
&= \hat{x}_k^- + K_k[y_k - h_k(\hat{x}_k^-, 0)] \\
P_k^+ &= (I - K_kH_k)P_k^- \qquad (13.43)
\end{aligned}$$

The discrete-time EKF can be summarized as follows.

The discrete-time extended Kalman filter

1. The system and measurement equations are given as follows:

$$\begin{aligned} x_k &= f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1}) \\ y_k &= h_k(x_k, v_k) \\ w_k &\sim (0, Q_k) \\ v_k &\sim (0, R_k) \end{aligned} \qquad (13.44)$$

2. Initialize the filter as follows:

$$\begin{aligned} \hat{x}_0^+ &= E(x_0) \\ P_0^+ &= E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \end{aligned} \qquad (13.45)$$

3. For $k = 1, 2, \cdots$, perform the following.

   (a) Compute the following partial derivative matrices:

$$\begin{aligned} F_{k-1} &= \left.\frac{\partial f_{k-1}}{\partial x}\right|_{\hat{x}_{k-1}^+} \\ L_{k-1} &= \left.\frac{\partial f_{k-1}}{\partial w}\right|_{\hat{x}_{k-1}^+} \end{aligned} \qquad (13.46)$$

   (b) Perform the time update of the state estimate and estimation-error covariance as follows:

$$\begin{aligned} P_k^- &= F_{k-1} P_{k-1}^+ F_{k-1}^T + L_{k-1} Q_{k-1} L_{k-1}^T \\ \hat{x}_k^- &= f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) \end{aligned} \qquad (13.47)$$

   (c) Compute the following partial derivative matrices:

$$\begin{aligned} H_k &= \left.\frac{\partial h_k}{\partial x}\right|_{\hat{x}_k^-} \\ M_k &= \left.\frac{\partial h_k}{\partial v}\right|_{\hat{x}_k^-} \end{aligned} \qquad (13.48)$$

   (d) Perform the measurement update of the state estimate and estimation-error covariance as follows:

$$\begin{aligned} K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + M_k R_k M_k^T)^{-1} \\ \hat{x}_k^+ &= \hat{x}_k^- + K_k[y_k - h_k(\hat{x}_k^-, 0)] \\ P_k^+ &= (I - K_k H_k) P_k^- \end{aligned} \qquad (13.49)$$

   Note that other equivalent expressions can be used for K_k and P_k^+, as is apparent from Equation (5.19).